Exercises EMGMT: Algorithmic Analysis

Note: All logarithms have base 2.

1. What is the sum of all even numbers from 0 to $2n$, for any integer $n > 0$?

2. Count the number of primitive operations that are performed in each of the two algorithms below.

   (a) **Algorithm** `arrayMax(A,n)`:
   Input: An array A storing $n \geq 1$ integers.
   Output: The maximum element in A.

   ```
   currentMax ← A[0]
   for i ← 1 to n − 1 do
       for j ← i to n − 1 do
           if A[i] > A[j] then
               if A[i] > currentMax then
                   currentMax ← A[i]
   return currentMax
   ```

   (b) **Algorithm** `weird(A,n)`:
   Input: An array A storing $n \geq 1$ integers.
   Output: An array F storing $n \geq 1$ integers.

   ```
   for i ← 1 to n do
       F[i] ← 0
       j ← i
       while j ≤ n do
           for k ← 0 to 4 do
               F[i] ← F[i] + A[j + k]
           j ← 2 · j
   return F
   ```

3. For each of the items below, find the smallest $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$:

   (a) n^3 is greater than $4n^2 + 60n$,

   (b) $8n\log(n)$ is smaller than $2n^2$,

   (c) 2^n is greater than n^4.

4. Give a $c > 0$ and an integer $n_0 \geq 1$ such that, for all $n \geq n_0$:

   (a) $40n^2 + 120 \leq cn^3$,

   (b) $16n\log(n^2) \leq cn^2$,

   (c) $\frac{1}{10}2^n \geq cn^4$.

5. Give a function $f(n)$ such that $22n^4 \log(n) + 130 \log^2(n)$ is $\Theta(f(n))$.

6. Order the following functions by asymptotic growth rate:

$4n\log(n) + 2n$	2^{10}	$2^{\log(n)}$
$3n + 100\log(n)$	$4n$	2^n
$n^2 + 10n$	n^3	$n\log(n)$

7. Prove:

   (a) $2n^3 + 9n^2$ is $O(n^3)$

   (b) $(n \log n)/8$ is $\Omega(n \log n)$

   (c) $2^{n+2} - n$ is $\Theta(2^n)$

8. Prove or disprove: if $d(n)$ is $O(f(n))$, then $ad(n)$ is $O(f(n))$, for any constant $a > 0$.

9. Prove or disprove: if $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$, then $d(n)-e(n)$ is $O(f(n) - g(n))$.

10. What are the running times of the algorithms in exercises 2(a) and 2(b) in big-Oh-notation?